

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)

B.Tech I Year I Semester Supplementary Examinations June-2024

ALGEBRA AND CALCULUS

(Common to all Branches)

Time: 3 Hours

Max. Marks: 60

(Answer all Five Units 5 x 12 = 60 Marks)

UNIT-I

- 1 a Reduce the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into Echelon form by CO1 L1 6M

using row transformations and find its rank.

- b Solve completely the system of homogeneous equations CO1 L1 6M
 $x + 2y + 3z = 0, 3x + 4y + 4z = 0, 7x + 10y + 12z = 0$

OR

- 2 Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy + 2xz - 2yz$ into CO1 L3 12M
the canonical form by Orthogonal transformation and discuss its nature.

UNIT-II

- 3 a Verify Rolle's Theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $(0, \pi)$ CO2 L4 6M
b Verify Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ CO2 L5 6M
in $[a, b]$

OR

- 4 a Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places CO2 L5 6M
using Taylor's theorem
b Obtain the Maclaurin's series expression for the function $f(x) = \cos x$ CO2 L6 6M

UNIT-III

- 5 a If $U = \log(x^3 + y^3 + z^3 - 3xyz)$ Prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) U = \frac{-9}{(x+y+z)^2}$. CO3 L5 6M
b If $u = \frac{x+y}{1-xy}$ and $\theta = \tan^{-1} x + \tan^{-1} y$, then find $\frac{\partial(u, \theta)}{\partial(x, y)}$. CO3 L1 6M

OR

- 6 a Show that the functions $u = xy + yz + zx, v = x^2 + y^2 + z^2$ and CO3 L1 6M
 $w = x + y + z$ are functionally related. Find the relation between them.
b By using Lagrange's undetermined multipliers, find the minimum CO3 L3 6M
value for $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

UNIT-IV

- 7 a Evaluate $\int_0^{\pi} \sin^8 \theta \cos^4 \theta d\theta$ CO4 L5 6M
b Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. CO4 L5 6M

OR

8 a Evaluate $\iint xy(x+y) dx dy$ over the region R bounded by $y = x^2$ and $y = x$. CO4 L5 6M

b Apply change of order of integration to evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$. CO4 L3 6M

UNIT-V

9 a Evaluate the integral $\int_0^1 x^2 \left[\log \left(\frac{1}{x} \right) \right]^3 dx$ CO5 L5 6M

b Prove that $\beta(m, n) = 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cdot \cos^{2n-1} \theta d\theta$ CO5 L3 6M

OR

10 a Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \frac{1}{5} \beta\left(\frac{2}{5}, \frac{1}{2}\right)$ CO5 L3 6M

b Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ CO5 L3 6M

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